Benson Example 5.9

Setup: masses m_1 and m_2 lie on the slope of the wedge, which makes an angle θ with respect to the horizontal. They are joined by a rope. The second mass m_2, which is higher than m_2, is in turn joined by a rope to m_3, which hangs off vertically the side of the wedge.

Erect the unit vectors $\widehat{i}'$ and $\widehat{j}'$ on the slope of the wedge – the $\widehat{i}'$ points upwards along it; whereas $\widehat{j}'$ is perpendicular to the former, but also pointing upward. The first mass has tension $T_1\widehat{i}'$, weight $-m_1 g \sin\theta \widehat{i}' - m_1 g \cos\theta \widehat{j}'$, and normal force $m_1 g \cos\theta \widehat{j}'$ acting upon it.

$$m_1 a = T_1 - m_1 g \sin\theta \tag{0.0.1}$$

The second mass has tension $-T_1\widehat{i}'$, tension $T_2\widehat{i}'$, weight $-m_2 g \sin\theta \widehat{i}' - m_2 g \cos\theta \widehat{j}'$, and normal force $m_2 g \cos\theta \widehat{j}'$ acting upon it.

$$m_2 a = T_2 - T_1 - m_2 g \sin\theta \tag{0.0.2}$$

Finally, we need to erect $\widehat{j}$ pointing vertically upwards; and $\widehat{i}$ perpendicular to it. Then the third mass has weight $-m_3 g \widehat{j}$ and tension $T_2\widehat{j}$ acting on it.

$$-m_3 a = T_2 - m_3 g. \tag{0.0.3}$$

Two remarks. We have assumed a common acceleration for all $m_{1,2,3}$; otherwise, the ropes will either break or go slack. For positive a, and say zero initial velocity, the masses $m_{1,2}$ are going to move up the slope; and m_3 will move downwards – hence, the $-$ sign in front of $m_3 a$.

We now add the m_1 and m_2 equations; and subtract the m_3 equation.

$$(m_1 + m_2 + m_3)a = -(m_1 + m_2) g \sin\theta + m_3 g \tag{0.0.4}$$

$$a = \frac{m_3 - (m_1 + m_2)\sin\theta}{m_1 + m_2 + m_3} g. \tag{0.0.5}$$