General Physics A: Midterm 2

1 Pushing a mass up a wedge

Consider a right-angle wedge of mass M, whose base is of length L and whose friction-less incline surface makes an angle θ with the horizontal, lying on a friction-less floor (which defines an inertial frame). At time $t = 0$, place a small mass $m < M$ at the bottom of the incline and exert a constant force F_0 perpendicular to and upon its vertical face; let the initial velocity of m with respect to the floor to be zero. Assume gravity $M\vec{g}$ acts upon the wedge; and $m\vec{g}$ upon the small mass m – both vertically downwards.

What is the threshold $F_0 = F_\star$ such that the small mass m would remain still *with respect to the wedge*? Next, suppose $F_0 > F_\star$ – determine the velocity of the small mass m with respect to the inertial frame when it reaches the top of the incline.

Note: The algebra may be messy, so be sure to highlight the key steps.

2 Nonlinear friction

Suppose the frictional force $\vec{F}_{\text{fric}}$ experienced by a mass m moving with velocity $\vec{v}$ inside a fluid is $\vec{F}_{\text{fric}} = -f|\vec{v}|\vec{v}$ and suppose the only other force acting on it is gravity (i.e., $m\vec{g}$). If the mass is released from rest at $t = 0$, integrate Newton's second law once to obtain

$$\vec{v}(t) = \sqrt{\frac{mg}{f}} \tanh\left(t\sqrt{\frac{fg}{m}}\right)\widehat{\jmath}, \tag{2.0.1}$$

where $\widehat{\jmath}$ is the unit vector pointing *towards* the Earth and $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$ is the hyperbolic tangent. What is the (terminal) velocity as $t \to \infty$? How could this terminal velocity be deduced without first integrating Newton's second law?

Denoting $\vec{v} = (\mathrm{d}y/\mathrm{d}t)\widehat{\jmath}$, and assuming $y(t = 0) = 0$, integrate eq. (2.0.1) once more to obtain

$$y(t) = \frac{m}{f} \ln\cosh\left(t\sqrt{\frac{fg}{m}}\right), \tag{2.0.2}$$

where $\cosh(x) = (e^x + e^{-x})/2$ is the hyperbolic cosine. Make a plot of this motion for $t \geq 0$. □