

General Physics A: Midterm 2

1 Pushing a mass up a wedge

Consider a right-angle wedge of mass M , whose base is of length L and whose incline surface makes an angle θ with the horizontal, lying on a friction-less floor (which defines an inertial frame). At time $t = 0$, place a small mass $m < M$ at the bottom of the incline and exert a constant force F_0 perpendicular to and upon its vertical face; let the initial velocity of m with respect to the floor to be zero. Assume gravity $M\vec{g}$ acts upon the wedge; and $m\vec{g}$ upon the small mass m – both vertically downwards.

What is the threshold $F_0 = F_*$ such that the small mass m would remain still *with respect to the wedge*? Next, suppose $F_0 > F_*$ – determine the velocity of the small mass m with respect to the inertial frame when it reaches the top of the incline.

Note: The algebra may be messy, so be sure to highlight the key steps.

2 Nonlinear friction

Suppose the frictional force \vec{F}_{fric} experienced by a mass m moving with velocity \vec{v} inside a fluid is $\vec{F}_{\text{fric}} = -f|\vec{v}|\vec{v}$ and suppose the only other force acting on it is gravity (i.e., $m\vec{g}$). If the mass is released from rest at $t = 0$, integrate Newton's second law once to obtain

$$\vec{v}(t) = \sqrt{\frac{mg}{f}} \tanh\left(t\sqrt{\frac{fg}{m}}\right) \hat{j}, \quad (2.0.1)$$

where \hat{j} is the unit vector pointing *towards* the Earth and $\tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$ is the hyperbolic tangent. What is the (terminal) velocity as $t \rightarrow \infty$? How could this terminal velocity be deduced without first integrating Newton's second law?

Denoting $\vec{v} = (dy/dt)\hat{j}$, and assuming $y(t = 0) = 0$, integrate eq. (2.0.1) once more to obtain

$$y(t) = \frac{m}{f} \ln \cosh\left(t\sqrt{\frac{fg}{m}}\right), \quad (2.0.2)$$

where $\cosh(x) = (e^x + e^{-x})/2$ is the hyperbolic cosine. Make a plot of this motion for $t \geq 0$. \square